Documentation of HyperHom-*SfePy*

1 Introduction

HyperHom-*SfePy* is a module for the general finite element solver *SfePy* (simple finite elements in Python, [1]) — a software written almost entirely in Python and is based on SciPy ([2]) and NumPy libraries.

This module can be used to solve the problems of numerical modelling of heterogeneous media undergoing large deformations. To derive the mathematical model we use the homogenization approach [3, 5, 6] which leads to the system of microscopic and macroscopic subproblems. At the microscopic scale we solve the problem of unknown corrector functions which are necessary for evaluation of homogenized material coefficients. These coefficients are employed at the macroscopic scale to solve a global response (displacements, pressure, ...).

We assume large deformations and a hyperelastic material model so the macro and micro subproblems can not be solved separately and we need a coupled micro-macro algorithm. The macroscopic (global) response is not homogenous in general case therefore the microscopic subproblems which provide the effective material parameters must be solved in whole macroscopic domain. This intrinsically increase the computational cost of our problem.

2 Mathematical models

2.1 Non-linear problem – Updated Lagrangian formulation

A heterogeneous hyperelastic material undergoing large deformations can be described by the incremental updated Lagrangian formulation (ULF) in which the discrete equations are formulated in the current configuration that is assumed to be the new reference configuration.

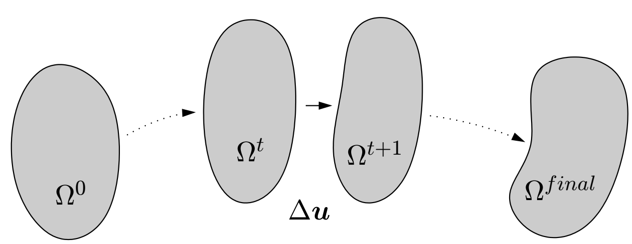


Figure 1: Incremental ULF algorithm.

2.2 Homogenization approach

This approach assumes a material microstructure as a periodic lattice generated by repetition of the so-called *reference periodic cell* (or representative volume element – RVE). The micromodel of a RVE is defined using the standard continuum theory. The homogenization machinery provides the *homogenized macroscopic constitutive parameters* as a result of the numerical solution of the local microscopic subproblem.

The governing macroscopic and microscopic equations for deforming media can be obtained in different ways. One possibility is to employ the standard *two-scale asymptotic* *method of homogenization* [9, 4], while another method is based on the *theory of unfolding operators*, as proposed in [3].

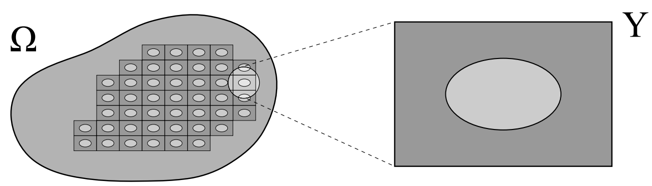


Figure 2: Macroscopic (Ω) and microscopic (Y) domains.

In order to determine the correct homogenized material parameters at a given macroscopic point, the corresponding local microstructure must reflect the macroscopic deformation and has to be in equilibrium. The updating process for the microstructure is driven by the relative macroscopic deformation, but it does not lead automatically to an equilibrated microscopic configuration, to find a correct microscopic configuration the iterative ULF algorithm must be used.

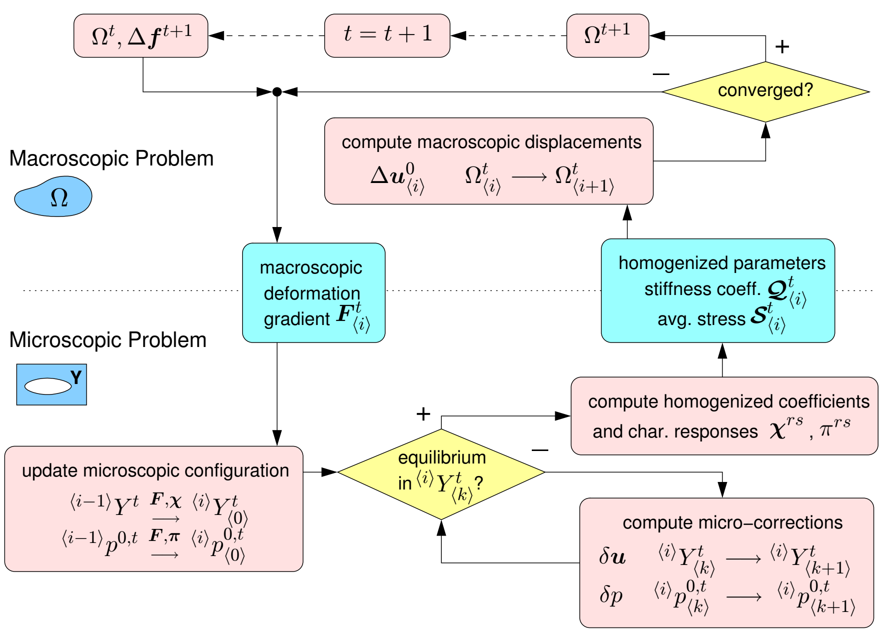
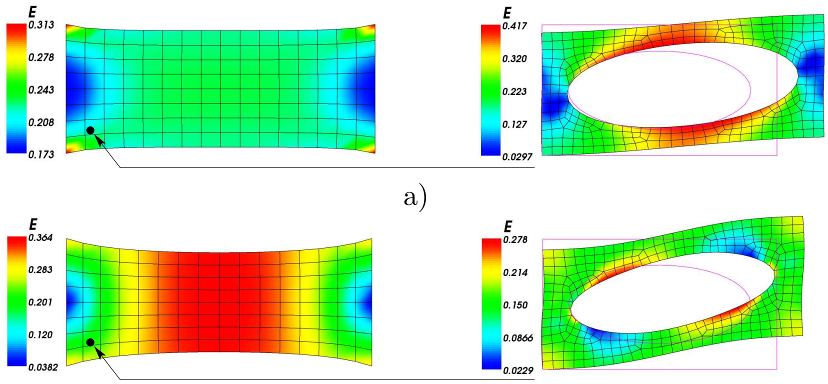


Figure 3: Macro–micro computational algorithm.

2.3 Numerical example

Two examples with different microstructures are shown: the microstructure having the a) void inclusion, b) incompressible inclusion. In Fig. 4 the strain distribution (norm of the strain tensor) in the deformed macroscopic domain and the local strains in the reference cell at a chosen macroscopic point are depicted.



a)

b)

Figure 4: Traction test – strain distribution (norm of the strain tensor) in the deformed macroscopic structure (left) and the microscopic strain in the deformed RVE at the marked macroscopic point (right); a) void inclusion, b) incompressible inclusion.

3 SfePy code

The source code, documentation and examples can be found at `http://code.google.com/p/sfepy/` and `http://docs.sfepy.org/doc/`. The weak forms (terms) required to solve the nonlinear coupled micro-macro simulations are defined mainly in `terms_hyperelastic_ul.py` and `termsHyperElasticity.c`.

References

[1] Cimrman, R. and others. SfePy: Simple Finite Elements in Python, http://sfepy.kme.zcu.cz, http://sfepy.org, 2009.

[2] http://www.scipy.org

[3] Cioranescu, D., Damlamian, A., Griso, G., 2008. The periodic unfolding method in homogenization. SIAM Journal on Mathematical Analysis 40 (4), 1585–1620.

[4] Cioranescu, D., Donato, P. An Introduction to Homogenization, Oxford Lecture Series in Mathematics and its Applications 17, Oxford University Press, Oxford, 1999.

[5] Griso G. and Rohan, E. On the homogenization of a diffusion-deformation problem in strongly heterogeneous media, Ricerche mat. (56) (2007) pp. 161–188.

[6] Hornung, U. Homogenization and Porous Media. Springer-Verlag, Berlin 1997.

[7] Rohan, E., Naili, S., Cimrman, R., Lemaire, T., 2009. Multiscale modelling of fluid saturated medium with double-porosity: application to the compact bone. Submitted.

[8] Rohan, E. Homogenization of the perfusion problem in dual-porous layer, Submitted.

[9] Sanchez-Palencia, E. Non-Homogeneous Media and Vibration Theory, Lecture Notes in Physics 127, Springer, Berlin, 1980.